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SHORTER COMMUNICATIONS

HEAT TRANSFER FROM A VERTICAL TRANSVERSELY VIBRATING PLANE SURFACE TO AIR BY FREE CONVECTION

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NOMENCLATURE

Mass [M], Length [L], Time [t], Temperature [θ] system of units.

- a, amplitude of vibration [L];
- C_p, constant pressure thermal capacity [$L^2/t^2\,\theta$];
- f, vibration frequency [1/t];
- Gr, Grashof number based on length of plate [dimensionless];
- $\bar{h}$, average convective heat-transfer coefficient [$M/t^3\,\theta$];
- k, thermal conductivity [$ML/t^3\,\theta$];
- L, length of vertical plate or plane surface [L];
- n, dimensionless constant;
- Re_{vib}, vibration Reynolds number [dimensionless], $= \frac{a_{RMS}\omega L}{\nu}$;
- S, mixing coefficient [L/t];
- $\bar{V}$, fluid velocity [L/t].

Greek symbols

- ν, kinematic viscosity [L^2/t];
- ρ, density [M/L^3];
- ω, circular frequency [1/t].

Subscripts

- v, pertaining to conditions with oscillation present;
- vo, pertaining to conditions at the critical vibration intensity;
- 0, pertaining to conditions in absence of oscillation;

Other subscripts defined in text.

THE OBJECT of this communication is to show how the Danckwerts–Mickley model for turbulent exchange may be applied to the above problem under conditions of transition to turbulence in the boundary layer.

Experiments [1, 2] with 6- and 8-in plates respectively, showed that at low intensities of vibration (intensity = af) there are small decreases in heat-transfer coefficient compared with that of the stationary plate, the boundary layer was observed to remain laminar. With increasing intensity a critical condition was observed at which transition occurred at the top of the plate, the turbulence being generated in the outer region of the boundary layer and propagating towards the surface. Intensities above the critical produced large increases in the heat-transfer coefficient.

Experiments [3, 4] on transition in free convective boundary layers on stationary plates, showed that turbulence originated in the region of the point of inflexion in the laminar velocity profile and propagated towards the surface. The fluid velocity at this point being 0·683 $\bar{V}_{max}$ for air; $\bar{V}_{max}$ being the maximum velocity in the laminar boundary layer at the point of transition.

From the Danckwerts–Mickley model [5] for the turbulent heat exchange between a forced fluid flow and a stationary surface.

$$\bar{h} = \sqrt{(k\rho C_p S)}. \tag{1}$$

The mixing coefficient S depends on a characteristic velocity and length of the system and the Reynolds number. For the problem under discussion here, with intensity greater than the critical, S will depend on a characteristic velocity $\bar{V}_v$ the length of the surface and the Grashof number. It has been found [6] for the case of superimposed acoustic vibration on turbulent flow, that S could be defined when the acoustic energy supplements the kinetic energy due to turbulence. Here it may be assumed that the vibration kinetic energy supplements the kinetic energy of the fluid in the critical layer so that

$$\bar{V}_v^2 = [(0{\cdot}683\ \bar{V}_{max})^2 + (a_{RMS}\omega)^2]. \tag{2}$$

In a manner similar to [5] but replacing the Reynolds number with Grashof number,

$$S \propto \frac{\bar{V}_v}{L}[Gr_v]^n. \tag{3}$$

If it is assumed that at the point of transition vibration does not supplement the energy of the boundary layer, then 0·683 $\bar{V}_{max}$ is substituted for $\bar{V}_v$ in (3). Also, at transition we know that $\bar{h}_{vo} \simeq \bar{h}_0$; hence from (1–3) and the foregoing argument

$$\frac{\bar{h}_v}{\bar{h}_0} = \left[1 + 2{\cdot}14\left(\frac{a_{RMS}\omega}{\bar{V}_{max}}\right)^2\right]^{\frac{1}{4}}. \quad (4)$$

If $\bar{V}_{max}$ is expressed in terms of the Grashof number [7] and transition is considered to take place at $x = L$ then

$$\bar{V}_{max} = 0{\cdot}55\frac{Gr_v^{\frac{1}{2}}\nu}{L}. \quad (5)$$

In the derivation of the foregoing equations it has been assumed that $\Delta\theta_v = \Delta\theta_0$, for comparison with experimental data [1, 2], obtained at constant heat flux, it is necessary to correct for this.

Substituting (5) in (4); correcting for temperature and defining a vibration Reynolds number we have

$$\frac{\bar{h}_v}{\bar{h}_0} = \left[1 + \frac{7{\cdot}09\,Re_{vib}^2}{Gr_v}\right]^{\frac{1}{4}}\left[\frac{Gr_v}{Gr_0}\right]^{\frac{1}{4}}. \quad (6)$$

Equation (6) with experimental data [1, 2] are shown in Fig. 1. The critical vibration Reynolds number will depend on the Grashof number for the static plate; the variation of $[Re_{vib}^2/Gr_0]_{critical}$ with Gr_0 from data [1, 2] is shown in Fig. 2. This gives the lower limit of validity of (6).

The upper limit of validity of (6) would occur when vibration forced convection controls; this can be said to occur when the heat-transfer coefficient for free convection at the centre of the plate is 10 per cent of that due to vibration.

For free convection the local coefficient at the centre of the plate $h_{L/2}$, [8] is

$$h_{L/2} \propto 0{\cdot}214\,Gr_0^{\frac{1}{4}}. \quad (7)$$

For vibration forced convection, assuming laminar flow conditions, h_S at stagnation point at centre of plate, [9] is

$$h_S \propto \left[\frac{a\omega L}{\nu}\right]^{\frac{1}{2}} \quad (8)$$

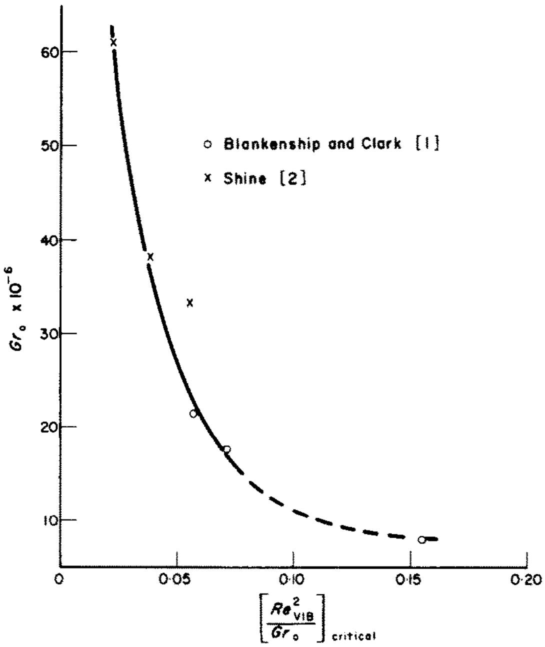

FIG. 2. Lower limit of $\frac{Re_{vib}^2}{Gr_0}$.

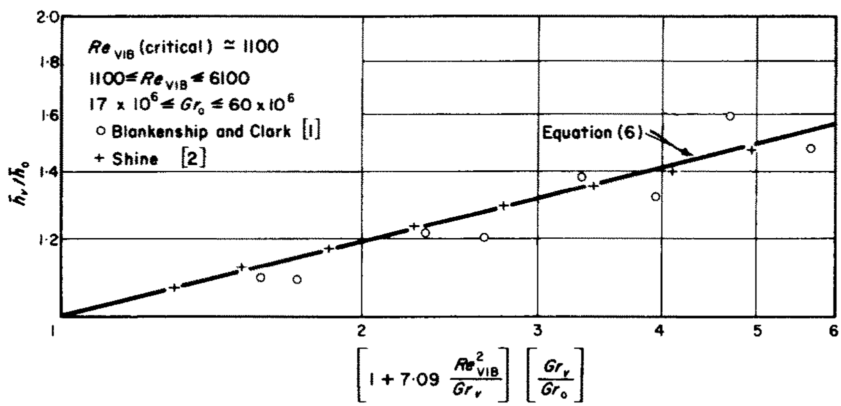

FIG. 1. Heat-transfer coefficient for a vertical transversely vibrating plane surface in free convection in air.

Hence

$$\frac{Re_{vib}^2}{Gr_0} \simeq 11. \qquad (9)$$

The range of data in [1] and [2] did not permit verification of (9). However, using arguments similar to above but modifying the equations for geometry [10], the data of [11] for free convection from horizontal cylinders executing large amplitude transverse vibrations gave an average value of $[Re_{vib}^2/Gr_0]_{forced} \simeq 80$ whereas prediction gave approximately 120, the discrepancy is to be expected because of the simplifying assumptions made including neglect of turbulence. This indicates that the right hand side of (9) should be approximately seven. The method discussed here would not be applicable for $Gr_0 \geqslant 1{\cdot}5 \times 10^8$ as a turbulent boundary layer would exist on the stationary plate.

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GENERALIZED FREE ENERGY METHOD FOR EQUILIBRIUM COMPOSITIONS IN COMPLEX MIXTURES

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KNOWLEDGE of thermodynamic and transport properties is essential for high temperature heat-transfer studies. In the process of property calculations the equilibrium composition of the reacting components must be determined. White, Johnson and Dantzig [1] have published a simple and effective free energy method for determining the equilibrium composition at a specified temperature and pressure. In this note we generalize the above method to include the condensed phase and electrically charged particles. Furthermore, the rate of change of the moles with respect to temperature is derived, a quantity used in specific heat calculations. Following White, Johnson and Dantzig let:

$$X = (x_1, x_2, \ldots x_n; x_1^0, x_2^0, \ldots x_n^0; x_1^1, x_2^1, \ldots x_n^1; \ldots x_1^N, x_2^N, \ldots x_n^N; x_e)$$

be the set of mole numbers where

x_j = moles of jth species of condensed phase;

x_j^0 = moles of jth species of neutral gas;

x_j^i = moles of jth species of i fold ionized gas;

x_e = moles of electrons.